RAN-2103000206020031

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## T.Y.B.Sc. (Sem. VI) Examination September - 2023

## Mathematics : Paper - MTH-601 : Ring Theory

**Time: 2 Hours ]** **[ Total Marks: 50**

**સૂચના : / Instructions**

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
**Fill up strictly the details of ☛ signs on your answer book**

Name of the Examination:
☛ **T.Y.B.Sc. (Sem. VI)**

Name of the Subject :
☛ **Mathematics : Paper - MTH-601 : Ring Theory**

Subject Code No.: **2103000206020031**

Seat No.:

Student's Signature

(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

**Q. 1 Answer the following as directed : (Any FIVE )** **(10)**

(1) Is the ring $\langle R = \{\overline{0}, \overline{4}, \overline{8}, \overline{12}, \overline{16}\}, +_{20}, \times_{20}\rangle$ with a Unit element? Justify your answer.

(2) Define Integral Domain. Give an example of commutative ring ; which is an integral domain but not a field.

(3) Let $\phi : R \to R \mid U$ be a homomorphism of a ring $R$ onto a Quotient ring $R \mid U$; defined by $\phi(a) = a + U$; for every $a \in R$. Find the Kernel $I(\phi)$ of a homomorphism $\phi$.

(4) Define: (i) Kernel of Homomorphism $\phi : R \to R'$ of a ring $R$ into a ring $R'$

(ii) Isomorphism.

(5) Express a greatest common divisor $\overline{4}$ of $\overline{2}$ and $\overline{5}$ as a linear combination of elements $\overline{2}$ and $\overline{5}$ in the Euclidean ring $J_7$ ; of integers modulo 7.

(6) Prove that $\overline{5}$ is also a greatest common divisor of the elements $\overline{4}$ and $\overline{6}$ in the commutative ring $J_7$ ; of integers modulo 7.


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(7) Justify : $\overline{6}$ and $\overline{9}$ are not relatively prime elements in the Euclidean ring $J_{11}$ ; of integers modulo 11.

(8) Mention all the prime elements in the Euclidean rings $\mathbb{Z}$ of all integers and $J_{23}$ ; of integers modulo 23.

**Q. 2 Answer any Two of the following: (10)**

(a) Define Field. In a ring $R$ with a *unit* element 1; prove that $a \cdot (-b) = -(a \cdot b)$ and $(-1) \cdot a = -a$ ; for all $a$, $b$ in $R$.

(b) Prove that every finite integral domain is a field.

(c) In a Boolean ring $R$; prove that
(i) $a + a = 0$; for every $a$ in $R$;
(ii) $a \cdot b = b \cdot a$; for all $a$, $b$ in $R$.

**Q. 3 Answer any Two of the following : (10)**

(a) Let $\phi : R \to R'$ be a homomorphism of a ring $R$ into a ring $R'$ ; with Kernel $I(\phi)$. Then prove that:
(i) $a, b \in I(\phi)$ implies $a - b \in I(\phi)$;
(ii) for every $a$ in $I(\phi)$ and $r$ in $R$ ; both $ra$ and $ar$ are in $I(\phi)$.

(b) (i) Mention all the ideals of the ring $\mathbb{Z}$ of all integers and the ring $J_{10}$ ; of integers modulo 10;
(ii) If $U$ is an ideal of a ring $R$ with a *unit* element 1 and $1 \in U$, then prove that $U = R$.

(c) Let a be a fixed element of a commutative ring $R$. Then prove that $r(a) = \{x \in R \mid a \cdot x = 0\}$ is a two – sided ideal of $R$.

**Q. 4 Answer any Two of the following : (10)**

(a) Define Euclidean Ring. Prove that every Euclidean ring possesses a unit element.

(b) Define *Unit* in a Commutative Ring with a *Unit* Element. Let $R$ be a Euclidean ring and $a \neq 0$, $b \neq 0$ in $R$. If $b$ is not *unit* in $R$, then prove that $d(a) < d(ab)$.

(c) Prove that the relation of "associates" in a commutative ring $R$ with a *unit* element is an equivalence relation on $R$.

**Q. 5 Answer any Two of the following : (10)**

(a) In a Euclidean ring $R$; prove that

(i) If $a$, $b$ and $c$ are elements in a Euclidean ring $R$ such that $(a, b) = 1$ and $a \mid bc$, then $a \mid c$;

(ii) If $\pi$ is a prime element in $R$ and $\pi \mid ab$; for $a, b \in R$, then; $\pi \mid a$ or ; $\pi \mid b$.

(b) Prove that an element a in *a* Euclidean ring $R$ is *unit* in $R$ if and only if $d(a) = d(1)$.

(c) If $A = (a_0)$ is a maximal ideal of a Euclidean ring $R$, then prove that $a_0$ is a prime element in $R$.

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